Symmetrices of square

Let us considerr a square ABCD with centre at O. There are eight symmetrices of the square—

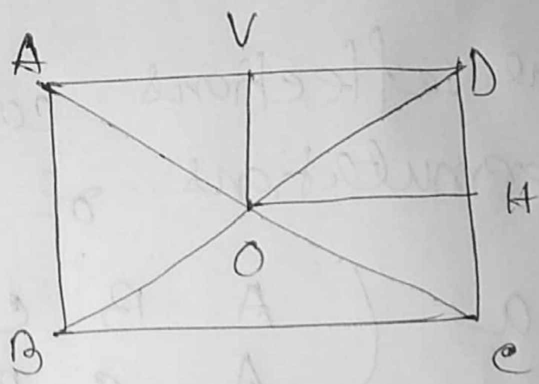
(1) Four rotations in the plane about O

i = rotation through 0°

r_1 = rotation through 90°

r_2 = rotation through 180°

r_3 = rotation through 270°



(2) Four rotations out of the plane

h = rotation about the horizontal plane OH,

v = " " " vertical " OV,

d = " " " (principal) diagonal OA

d' = " " " (other) " OB.

Let 'o' stands for the composition of mappings.

Let $S = \{ i, r_1, r_2, r_3, h, v, d, d' \}$. Taking as the binary composition on the set S, the composition table is given below :—

$\circ$	i	r_1	r_2	r_3	h	v	d	d'
i	i	r_1	r_2	r_3	h	v	d	d'
r_1	r_1	r_2	r_3	i	d'	d	h	v
r_2	r_2	r_3	i	r_1	v	h	d'	d
r_3	r_3	i	r_1	r_2	d	d'	v	h
h	h	d	v	d'	i	r_2	r_1	r_3
v	v	d'	h	d	r_2	i	r_3	r_1
d	d	v	d'	h	r_3	r_1	i	r_2
d'	d'	h	d	v	r_1	r_3	r_2	i

The eight symmetrices of the square form a non commutative group [since the composition table is not symmetric about its principal diagonal]. This group is called the octic group or the dihedral group D_4.

Note:- ① The symmetrices correspond to the following permutations of the ~~ver~~ vertices —

$$i = \begin{pmatrix} A & B & C & D \\ A & B & C & D \end{pmatrix};\quad r_1 = \begin{pmatrix} A & B & C & D \\ B & C & D & A \end{pmatrix};\quad r_2 = \begin{pmatrix} A & B & C & D \\ C & D & A & B \end{pmatrix}$$

$$r_3 = \begin{pmatrix} A & B & C & D \\ D & A & B & C \end{pmatrix};\quad h = \begin{pmatrix} A & B & C & D \\ B & A & D & C \end{pmatrix};\quad v = \begin{pmatrix} A & B & C & D \\ D & C & B & A \end{pmatrix}$$

$$d = \begin{pmatrix} A & B & C & D \\ A & D & C & B \end{pmatrix};\quad d' = \begin{pmatrix} A & B & C & D \\ C & B & A & D \end{pmatrix}$$

② ~~The~~ The symmetrices of a regular n-gon form a non commutative group of order $2n$, called the dihedral group D_n.